

Reasoning with Properties from Algebra

- Goals**
- Use properties from algebra.
 - Use properties of length and measure to justify segment and angle relationships.

ALGEBRAIC PROPERTIES OF EQUALITY

Let a , b , and c be real numbers.

Addition Property	If $a = b$, then $a + c = b + c$
Subtraction Property	If $a = b$, then $a - c = b - c$
Multiplication Property	If $a = b$, then $ac = bc$
Division Property	If $a = b$ and $c \neq 0$, then $\frac{a}{c} = \frac{b}{c}$
Reflexive Property	For any real number a , $a = a$
Symmetric Property	If $a = b$, then $b = a$
Transitive Property	If $a = b$ and $b = c$, then $a = c$
Substitution Property	If $a = b$, then a can be substituted for b in any equation or expression

Example 1 Writing Reasons

Solve $-2x + 1 = 56 - 3x$ and write a reason for each step.

$$-2x + 1 = 56 - 3x$$

Given

$$X + 1 = 56$$

add prop of equality

$$x = 55$$

subtract prop of equality

✓ **Checkpoint** Solve the equation. Write a reason for each step.

1. $12x - 3(x + 7) = 8x -$ given

$-3(x + 7) = -4x$ - subtract prop of equality

$-3x - 21 = -4x$ - distributive prop of equality

$-21 = -x$ - addition prop of equality

$21 = x$ - division prop of equality

Example 2 Using Properties in Real Life

Science The Fahrenheit and Celsius temperature scales are related by the formula $F = \frac{9}{5}C + 32$, where F represents degrees Fahrenheit and C represents degrees Celsius.

- Solve the formula for C and write a reason for each step.
- Use the result to find the Celsius temperatures that correspond to the following Fahrenheit temperatures: 5°F , 32°F , 95°F , 140°F , 212°F . How does the Celsius temperature change as the Fahrenheit temperature changes?

Solution

a. $F = \frac{9}{5}C + 32$ Given

$$F - 32 = \frac{9}{5}C \quad \text{Subtraction property of equality}$$

$$\frac{5}{9}(F - 32) = C \quad \text{Multiplication property of equality}$$

- b. Use substitution to find the Celsius temperature that corresponds to 5°F .

$$\frac{5}{9}(F - 32) = C \quad \text{Given}$$

$$F = 5$$

$$\frac{5}{9}(5 - 32) = C \quad \text{Substitution property of equality}$$

$$-15 = C \quad \text{Simplify.}$$

Find the other corresponding temperatures using the same method.

Temperature ($^{\circ}\text{F}$)	5	32	95	140	212
Temperature ($^{\circ}\text{C}$)	-15	0	35	60	100

From the table, you can see that the Celsius temperature increase as the Fahrenheit temperature increase.

PROPERTIES OF EQUALITY

	Segment Length	Angle Measure
Reflexive	For any segment AB, <u>$AB = AB$</u> .	For any angle A, <u>$m\angle A = m\angle A$</u>
Symmetric	If $AB = CD$, then <u>$CD = AB$</u> .	If $m\angle A = m\angle B$, then <u>$m\angle B = m\angle A$</u> .
Transitive	If $AB = CD$ and $CD = EF$, then <u>$AB = EF$</u> .	If $m\angle A = m\angle B$ and $m\angle B = m\angle C$, then <u>$m\angle A = m\angle C$</u> .

Example 3 Using Properties of Measure

Use the information at the right to find $m\angle 1$.

$$\begin{aligned} m\angle 1 + m\angle 2 + m\angle 3 + m\angle 4 &= 360^\circ \\ m\angle 2 + m\angle 3 &= m\angle 4 \\ \hline m\angle 1 &= m\angle 4 \end{aligned}$$

Solution

$$m\angle 1 + m\angle 2 + m\angle 3 + m\angle 4 = 360^\circ$$

$$m\angle 2 + m\angle 3 = m\angle 4$$

$$m\angle 1 + m\angle 4 + m\angle 4 = 360^\circ$$

$$3(m\angle 4) = 360^\circ$$

$$\frac{3}{3} m\angle 4 = \frac{360^\circ}{3}$$

$$m\angle 1 = 120$$

Given

Given

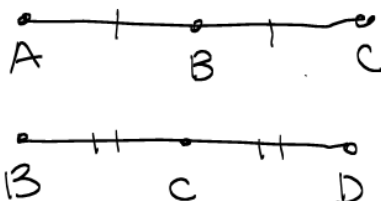
Given

Substitution property of equality

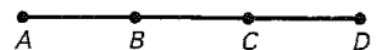
Simplify.

Division property of equality

Transitive property of equality



2. In the diagram at the right, B is the midpoint of AC and C is the midpoint of BD. Show that $AB = CD$.



B is midpoint of AC - given

C is midpoint of BD - given

$AB = BC$ - def of midpoint

$BC = CD$ - def of midpoint

$AB = CD$ - transitive prop